Brownian motion and some of its applications to Fluid Dynamics

Ana Bela Cruzeiro
Grupo de Física-Matemática da UL
and
Dep. de Matemática do IST
ana.cruzeiro@tecnico.ulisboa.pt

Abstract

I will present the definition and a little bit of history of the Brownian motion ([2], [5]). This stochastic process has applications in many areas, I will focus on some in Fluid Dynamics. More precisely, I will describe a stochastic Lagrangian approach to the Navier-Stokes equation that generalizes the one formulated by Vladimir Arnold for the Euler equation ([1]). The Lagrangian Euler paths are, according to Arnold, geodesics on a Lie group of diffeomorphisms. In our approach, initiated in [4], the Lagrangian paths are random but the velocity (interpreted in an adequate sense) also satisfy a minimal action principle. This approach has been developed in various directions, allowing to cover many dissipative systems and to include, in particular, advected quantities ([3]).

References

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