Book Of Abstracts

Editors:
P. Serranho
F. Cipriano
G. Ferreira
C. Gonçalves
M.R. Grossinho
M.R. Oliveira
M.R. Ramos

UNIVERSIDADE
AbERTA
www.uab.pt

List of Invited Talks

List of Contributed Talks

Invited Talk 1

The Sleeping Beauty

João Araújo[1,*]
[1] NOVA Faculty of Sciences and Technology
*jj.araujo@fct.unl.pt

Abstract

The goal of this talk is to show how the prince of Augmented Intelligence Theorem Proving (AgITP) is awakening the sleeping beauty of mathematics. [1], [2] and [3].

We will explore the transformative potential of Augmented Intelligence Theorem Proving (AgITP) in the realm of mathematics. The concept of AgITP can be seen as the prince that is awakening the sleeping beauty of mathematics by unlocking new and innovative ways to approach mathematical problems and enhance human understanding.

We will delve into the following topics:

1. Introduction to Augmented Intelligence Theorem Proving (AgITP): A brief overview of AgITP, its origins, and its current state in the field of mathematics and computer science.
2. The impact of AgITP on mathematical discovery: We will discuss how AgITP is fostering collaboration between humans and machines to generate novel insights, solve complex problems, and potentially revolutionize the way mathematics is conducted.
3. The role of AgITP in improving mathematical education: By leveraging the capabilities of AgITP, we will explore how educators can enhance the learning experience for students and promote deeper understanding of mathematical concepts.
4. Real-world applications of AgITP in mathematics: We will present examples of successful applications of AgITP in various branches of mathematics, demonstrating its potential to contribute to groundbreaking discoveries, including a proof with one million pages and the solution of $\aleph_0$ problems, open since the 70s.
5. Challenges and future directions in AgITP: Lastly, we will address the current challenges faced by AgITP, as well as the opportunities and potential developments in the field that could further enhance its impact on mathematics.

Through this talk, we aim to demonstrate the transformative power of AgITP as it awakens the dormant potential of mathematics, paving the way for a new era of mathematical discovery and understanding.

References

[1] www.proverx.com

[2] https://sgv.pythonanywhere.com

[3] http://fst.educa.pt

Invited Talk 2

Brownian motion and some of its applications to Fluid Dynamics

Ana Bela Cruzeiro[1,2,*]
[1]Grupo de Física-Matemática da UL
[2] Dep. de Matemática do IST
[*]ana.cruzeiro@tecnico.ulisboa.pt

Abstract

I will present the definition and a little bit of history of the Brownian motion ([2], [5]). This stochastic process has applications in many areas, I will focus on some in Fluid Dynamics. More precisely, I will describe a stochastic Lagrangian approach to the Navier-Stokes equation that generalizes the one formulated by Vladimir Arnold for the Euler equation ([1]). The Lagrangian Euler paths are, according to Arnold, geodesics on a Lie group of diffeomorphisms. In our approach, initiated in [4], the Lagrangian paths are random but the velocity (interpreted in an adequate sense) also satisfy a minimal action principle. This approach has been developed in various directions, allowing to cover many dissipative systems and to include, in particular, advected quantities ([3]).

References

[1] Arnold, V.I. "Sur la géométrie différentielle des groupes de Lie de dimension infinie et ses applications a l'hidrodynamique des fluides parfaits," Ann. Inst. Fourier 16,(1966), pp. 316–361

[2] Brown, R. "A brief account of microscopical observations made in the months of June, July and August, 1827, on the particles contained in the pollen of plants; and on the general existence of active molecules in organic and inorganic bodies ," The Philosophical Magazine and Annals of Philosophy.,(1828).

[3] Chen, X., Cruzeiro, A.B., Ratiu, T.S. "Stochastic Variational Principles for Dissipative Equations with Advected Quantities," J. Nonlinear. Sci. 33,(2023), Paper n.5

[4] Cipriano, F., Cruzeiro, A.B. "Navier-Stokes equation and diffusions on the group of homeomorphisms of the torus," Comm. Math. Phys. 275,(2007), pp. 255–269

[5] Nelson, E. Dynamical Theories of Brownian motion, Princeton Univ. Press (1967).

Invited Talk 3

On a mathematical curve that shapes the world: From WW2 to generative AI

Miguel de Carvalho[1,*]
[1] UNIVERSITY *of* EDINBURGH
[*] Miguel.deCarvalho@ed.ac.uk

Abstract

In this talk, I will offer a quick overview on the history of the ROC (Receiver Operating Characteristic) curve, along with a biased personal account of some selected developments on the topic that I have made with co-authors so to define priors on the space of ROC curves. Roughly speaking, the ROC curve is a mathematical and statistical object that can be used for comparing two probability measures so to assess the degree of overlap between their density functions. A prior on the space of ROC curves can be defined by resorting to pairs of random probability measures, which can then be used for learning about ROC curves from data by resorting to Bayes theorem. Regression versions of this approach can be devised, so to define priors on the space of conditional ROC curves.

The historical outlook will showcase that the ROC curve was born out of World War II—for detecting enemy objects in battlefields—and that it was fundamental over the COVID 19 pandemic—for examining the accuracy of COVID tests.

Finally, I will make some remarks on how ROC curves are being fundamental at the moment in the field of generative Artificial Intelligence (AI), for answering questions such as:

"Was this abstract written by a human or by AI?"

References

[1] de Carvalho M., Barney B. J., Page, G. L. (2020) "Affinity-based measures of biomarker performance evaluation," Statistical Methods in Medical Research, **29**, No. 3, 837–853 (2020).

[2] Inácio de Carvalho V., de Carvalho M., Branscum A. J. "Bayesian Bootstrap Inference for the ROC Surface," Stat, **7**, e211 (2018).

[3] Inácio de Carvalho V., de Carvalho M., Branscum A. J. "Nonparametric Bayesian covariate-adjusted estimation of the Youden index," Biometrics, **73**, No. 4, 1279–1288 (2017).

[4] de Carvalho M., Page G. L., Barney, B. "On the geometry of Bayesian inference," Bayesian Analysis, **14**, No. 4, 1013–1036 (2019).

[5] Inácio de Carvalho V., Jara A. E., Hanson, T., de Carvalho M. "Bayesian nonparametric ROC regression modeling ," Bayesian Analysis, **73**, No. 8, 1279–1288 (2013).

Invited Talk 4

Persistence and permanence for nonautonomous delay differential systems

Teresa Faria[1,*]
[1] University of Lisbon,
[*]teresa.faria@fc.ul.pt

Abstract

The global dynamics of a family of nonautonomous systems of delay differential equations is studied. This family includes structured systems inspired in mathematical biology models, with either discrete or distributed delays in both the linear and nonlinear terms. Sufficient conditions for the persistence and permanence are established [1]. For periodic systems, criteria for the existence of positive periodic solutions are also given [2]. The results are illustrated with applications.

References

[1] T. Faria, "Permanence for nonautonomous differential systems with delays in the linear and nonlinear terms," Mathematics, **9**, No. 263, 1–20 (2021).

[2] T. Faria, R. Figueroa, "Positive periodic solutions for systems of impulsive delay differential equations," Discrete Contin. Dyn. Syst. Ser. B, **28**, 170–196 (2023).

Contributed Talk 1

Analysis of crew time series absenteeism in the railway sector

Catarina, Afonso[1*], Ricardo Saldanha[1,2], Regina Bispo[3]
[1]NOVA School of Science and Technology, NOVA University of Lisbon
[2] SISCOG - Sistemas Cognitivos, SA
[3]Center for Mathematics and Applications (NOVA Math) and Department of Mathematics, NOVA School of Science and Technology, NOVA University of Lisbon
[*]cmr.afonso@campus.fct.unl.pt

Abstract

Crew absenteeism is a problem for railway operators because it can have repercussions, such as cancellation of train trips, additional expenses with reserve manpower for handling unpredicted situations, with overtime payments, among others.

To maintain the normal operation of transport, railway operators carry out operational planning to manage the railway resources, namely, the railroad, vehicles and crew members. However, if an unexpected event occurs, like crew unplanned absence from work, the operational planning has to be readjusted as soon as possible. As some types of absences of crew members, particularly those caused by illness or family emergencies, are not planned, the railway operators seek decision support tools that can help mitigate the impacts caused by absenteeism, to conform the planning to operational needs.

Hence, the aim of this study is to develop a prediction model for unscheduled absences that can estimate the percentage of crew members absent, segmented by job title (drivers or guards) and by operational base, for a given time period based on historical data. This prediction model can help planners and dispatchers take better decisions, for instance, in creating reserve plans with increased cost-effectiveness.

To achieve this goal, several datasets with attributes relevant to the problem were selected and exported from a railway operator in Northern Europe, a customer of SISCOG[†].

The data was converted to time series in order to enable the analysis of the evolution of absenteeism over time, in this case, daily data on the percentage of absence by type of function performed and operational base. By using forecasting methods, we looked for patterns and analyzed which factors significantly influence absenteeism. In particular, we used time series models as, e.g., SARIMA and SARIMAX models to predict absenteeism.

[†]The company that hosts this research and develops decision-support tools for the scheduling and managment of railway operations, such as crew operation

Contributed Talk 2

NLS Bound States on $\mathcal{T}$ graphs

Francisco Agostinho[1,2,*], Hugo Tavares[1,2], Simão Correia[1,2]
[1] Centro de Análise Matemática, Geometria e Sistemas Dinâmicos,
[2] Instituto Superior Técnico, Av. Rovisco Pais, Lisboa Portugal
[*] fcagostinho95@gmail.com

Abstract

In this talk we focus on the Nonlinear Schrödinger Equation (NLS) with a pure power nonlinearity defined on a metric graph.

$$-u'' + \lambda u = |u|^{p-2}u, \text{ in } \mathcal{G}_\ell. \tag{1}$$

The metric graph in question, $\mathcal{G}_\ell$, is a $\mathcal{T}$ graph and it is composed of two half-lines and a terminal edge, of length $\ell > 0$, all joined at the same vertex.

We present a characterization of the (positive) bound states of the equation (1) in the $\mathcal{T}$ graph and we analyze their dependece on the parameter ℓ.

A physically relevant class of bound states are the so called *ground states*. For the case of (1), these bound states can be obtained via variational mehtods. We exhibit two possible variational approaches: the energy variational approach; and the action variational approach. Each approach has its own set of solutions. We show that, as a consequence of the previous characterization, (action) ground states are in fact unique. We then compare this (action) ground state with the solutions of the energy variational approach and we show that for subcritical values of the nonlinearity, that is for $p < 6$, if p sufficiently close to 6 both sets of solutions are distinct.

References

[1] Agostinho. F. "Existence and qualitative properties of solutions to nonlinear Schrödinger equations on metric graphs" MSc Thesis (2020); http://hdl.handle.net/10451/45439.

[2] Dovetta. S. Serra. E. Tilli. P. 'Action versus energy ground states in nonlinear Schrödinger equations" Math. Ann. (2022)

Contributed Talk 3

A glimpse into the fully nonlinear mean-field games systems

Pêdra Andrade[1,*], Julio Correa[2]
[1] Instituto Superior Técnico - Universidade de Lisboa
[2] Instituto de Matemática e Estatística - Universidade Estadual do Rio de Janeiro
[*]pedra.andrade@tecnico.ulisboa.pt

Abstract

We discuss recent advances in fully nonlinear mean-field game systems. Based on the techniques in [2, 3] and the ideas in [1], we are able to obtain improved regularity for the solutions in Sobolev spaces, when the right-hand side is a discontinuous function. In addition, we prove the existence of minimizers for the variational formulation and the existence of solutions to the mean-field games system.

References

[1] Andrade, Pêdra. and Pimentel, Edgard. "Stationary fully nonlinear mean-field games," in the journal, J. Anal. Math., **145**, No. 1, 335–356, (2021).

[2] Fabes, Eugene. and Stroock, Daniel. "The L^p-integrability of Green's functions and fundamental solutions for elliptic and parabolic equations" in the journal, Duke Math. J., **51**, No. 4,997–1016 (1984).

[3] Caffarelli, L. and Crandall, M. and Kocan, M. and Swiech, A. "On viscosity solutions of fully nonlinear equations with measurable ingredients" in the journal, Comm. Pure Appl. Math., **49**, No. 4, 365–397 (1996).

Contributed Talk 4

Improving the accuracy and interpretability of linear models using cumulants

Patrícia Antunes[1,*], Sandra S. Ferreira[2],
Dário Ferreira[2],
[1] Centre of Mathematics and Applications, University of Beira Interior, Portugal
[2] Department of Mathematics, Centre of Mathematics and Applications, University of Beira Interior, Portugal
[*] antunes.patricia03@gmail.com

Abstract

This article presents an overview of the use of fourth cumulants in linear models. Fourth cumulants are useful for extracting additional information from statistical models and can be a powerful tool for understanding the data. We discuss the definition of these cumulants and describe how they can be used to identify patterns and relationships in data, as well as to assess the fit of linear models. We also provide an example using fourth cumulants in linear models.

References

[1] Antunes P., Ferreira S. S.,Ferreira D., Nunes C. & Mexia J. T. , "Estimation in additive models and ANOVA-like applications," Journal of Applied Statistics, **47**, 13–15 (2020).

[2] Balakrishnan,N., Johnson, N. L. & Kotz, S., "A note on relationships between moments, central moments and cumulants from multivariate distributions," Statistics & Probability Letters, **39**, 1, 49–54 (1998).

[3] Demidenko, E, Mixed Models: Theory and Applications, John Wiley & Sons, New Jersey (2004).

[4] Johnson, N. L., Kotz, S. & Balakrishnan, N., Continuous Univariate Distributions, 2nd edition, vol 1, John Wiley, New Jersey (1994).

[5] Koutrouvelis, I. A., Canavos, G. C. & Kallioras, A. G., "Cumulants Plots for Assessing the Gamma Distribution," Communications in Statistics-Theory and Methods, **39**, 4, 626–641 (2010).

[6] Lehmann, E. L. Testing Statistical Hypotheses, 2nd edition, John Wiley, New Jersey (1986).

[7] Zheng, Q., "Computing relations between moments and cumulants," Computational Statistics, **17**, 4, 507–515 (2002).

Contributed Talk 5

On a partial differential equation driven by white and fractional noises

Neeraj Bhauryal[1,a], Pierre-A. Vuillermot[2,3,b]
[1] Grupo de Física Matemática, University of Lisbon, Portugal
[2] Grupo de Física Matemática, University of Lisbon, Portugal [3] Inst. Élie Cartan de Lorraine, Nancy, France
[a] nsbhauryal@fc.ul.pt . , [b] pierre.vuillermot@univ-lorraine.fr

Abstract

We study existence-uniqueness of variational solutions for a class of parabolic stochastic partial differential equations

$$du(x,t) = (\mathrm{div}(k(x,t)\nabla u(x,t)) + f(u(x,t)))\mathrm{d}t + g(u(x,t))W(x,\mathrm{d}t) + h(u(x,t))W^H(x,\mathrm{d}t),$$

defined on $\mathbb{R}^d \times (0,T)$ with $T > 0$ fixed and $u(0,x) = u_0(x)$. Here W and W^H are standard and fractional $L^2(\mathbb{R}^d)$-valued Brownian motions respectively. This talk is based on an ongoing work.

Contributed Talk 6

Temperature-Mortality Association: Portuguese Extreme Weather Event Early Warning System

André Brito[1,2,*], Baltazar Nunes [2,3], Susana Silva [2], Regina Bispo [1,4]
[1] NOVA School of Science and Technology, Universidade NOVA de Lisboa
[2] Department of Epidemiology, Instituto Nacional de Saúde Dr Ricardo Jorge, Lisboa, Portugal
[3] Centro de Investigação em Saúde Pública, Escola Nacional de Saúde Pública, Universidade NOVA de Lisboa, Lisboa, Portugal
[4] Center for Mathematics and Applications (NOVA Math)
[*] anm.brito@campus.fct.unl.pt

Abstract

Portugal is among the European countries with higher excess of mortality during winter, even though winters are considered to be relatively mild. There is also an excess of mortality during the summer. This excess mortality might be associated with a larger vulnerability of the Portuguese population to non-optimal temperature exposure caused by poor housing conditions and an ageing population.

In this study, supported by FCT (RELIABLE: DSAIPA/DS/0111/2019), an update of the heat and cold health early warning systems is proposed for use in Mainland Portugal. The aim was to develop a risk indicator, active throughout the whole year, and easily understood by the entire population, with the highest possible spatial resolution. Daily data of all-causes mortality, maximum and minimum temperatures were gathered from public data sources for the 1995-2020 time period. District-specific temperature-mortality associations were estimated using quasi-Poisson regression. Linear threshold Distributed Lag Models (DLM) were proposed and estimated for cold and warm semesters, where minimum temperatures were considered in autumn/winter and maximum temperatures in spring/summer, to identify worst case exposure scenarios. Influenza incidence was also included in the models to improve predictive performance. Model selection was based on goodness-of-fit criteria.

Models proposed here could serve as updates for heat and cold health early warning systems, as they provide the results to maintain a risk indicator with the aforementioned properties. Differences between the optimum district-specific models were found and completely justify the need for region-specific warnings. Optimum cold thresholds were found to be relatively mild temperatures when compared to optimum heat thresholds, suggesting the effects of cold temperatures on mortality start at fairly milder temperatures. Evidence was found supporting the hypothesis of acclimatisation of the population to their own specific climates.

References

[1] Bhaskaran, K., Gasparrini, A., Hajat, S., Smeeth, L. & Armstrong, B. Time series regression studies in environmental epidemiology. *International Journal Of Epidemiology.* **42**, 1187-1195 (2013)

[2] Guo, Y., Gasparrini, A., Armstrong, B., Tawatsupa, B., Tobias, A., Lavigne, E., De Sousa Zanotti Stagliorio Coelho, M., Pan, X., Kim, H., Hashizume, M., Honda, Y., Leon Guo, Y., Wu, C., Zanobetti, A., Schwartz, J., Bell, M., Scortichini, M., Michelozzi, P., Punnasiri, K., Li, S., Tian, L., Garcia, S., Seposo, X., Overcenco, A., Zeka, A., Goodman, P., Dang, T., Van Dung, D., Mayvaneh, F., Saldiva, P., Williams, G. & Tong, S. Heat wave and mortality: A multicountry, multicommunity study. *Environmental Health Perspectives*. **125**, 1-11 (2017)

[3] Healy, J. Excess winter mortality in Europe: A cross country analysis identifying key risk factors. *Journal Of Epidemiology And Community Health*. **57**, 784-789 (2003)

[4] Gasparrini, A., Masselot, P., Scortichini, M., Schneider, R., Mistry, M., Sera, F., Macintyre, H., Phalkey, R. & Vicedo-Cabrera, A. Small-area assessment of temperature-related mortality risks in England and Wales: a case time series analysis. *The Lancet Planetary Health*. **6**, e557-e564 (2022), http://dx.doi.org/10.1016/S2542-5196(22)00138-3

[5] Gasparrini, A. Modeling exposure-lag-response associations with distributed lag non-linear models. *Statistics In Medicine*. **33**, 881-899 (2014)

Contributed Talk 7

Unilateral Problems for Quasilinear Operators with Fractional Riesz Gradients

Pedro Campos[1,2,*], José Francisco Rofrigues[1,2]

[1] Centro de Matemática, Aplicações Fundamentais e Investigação Operational,
[2] Departamento de Matemática, Faculdade de Ciências, Universidade de Lisboa P-1749-016 Lisboa, Portugal
[*] pmcampos@fc.ul.pt

Abstract

In recent years there has been an interest in the study of fractional partial differential equations due to their applications to the modeling of non-local phenomena.

In this talk, we consider a generalization of divergent form partial differential equations to variational inequalities

$$\int_{\mathbb{R}^d} a(x,u,D^s u)) \cdot D^s(v-u) + b(x,u)(v-u)\,dx \geq \langle L, v-u\rangle \quad \forall v \in K, \tag{2}$$

where $\Lambda_0^{s,p}(\Omega)$ is a fractional Sobolev type space called the Lions-Calderón space, D^s is the Riesz fractional gradient (for short, fractional gradient) as defined in [3, 4], and $K \subset \Lambda_0^{s,p}(\Omega)$ is a non-empty, closed and convex set.

Besides the usual existence and uniqueness results, we also present some results regarding the continuous dependence of the solutions of (2) when the parameter $s \in (0,1)$ changes. In particular, we are going to see some examples where sequences of solutions to (2) converge to a solution of another variational inequality, as $s \to \sigma \in [0,1]$.

We conclude the talk with some general results of existence of solutions of quasi-variational inequalities, i.e., problems of the form of (2) where the convex set K depends on u, as well as applications to particular problems.

References

[1] Bruè Elia, Calzi Mattia, Comi Giovanni E. and Stefani Giorgio "A distributional approach to fractional Sobolev spaces and fractional variation: asymptotics II," C. R. Math. Acad. Sci. Paris, **360**, 589–626 (2022).

[2] Campos, P., "Lions-Calderón spaces and applications to nonlinear fractional partial differential equations," MSc Thesis (2021): http://hdl.handle.net/10451/52753.

[3] Shieh, T.-T. and Spector, D., "On a new class of fractional partial differential equations," Adv. Calc. Var., **8**, No. 4, 321–336 (2015).

[4] Šilhavý M. "Fractional vector analysis based on invariance requirements (critique of coordinate approaches)," Contin. Mech. Thermodyn., **32**, No. 1, 207–228 (2020).

Contributed Talk 8

Comparative Analysis of Formulations for the Hamiltonian p-Median Problem

Francisco Canas[1,*], Luís Gouveia[1]
[1] Faculdade de Ciências, Universidade de Lisboa,
[*]francisco.m.p.canas@gmail.com

Abstract

We introduce and study new compact formulations for the symmetric Hamiltonian p-Median Problem (HpMP). Given a weighted complete undirected graph $G = (V, E)$ with weights on the edges and a positive integer p, the HpMP on G is to find a minimum weight set of p elementary cycles partitioning the vertices of G. Compact formulations have the advantage of being convenient and easily used in combination with off-the-shelf optimization software, unlike other types of formulations which may consider exponentially sized sets of variables or constraints. We focus more on compact formulations for eliminating solutions with less than p cycles, as such formulations are less well known than formulations which prevent solutions with more than p cycles.

The proposed formulations consider integer variables that assign labels to nodes and prevent less than p cycles by stating that different depots must have different labels and that nodes in the same cycle must have the same label. The main difference between the two classes of formulations lies in the constraints that set the values of the labels of the depots, and as a consequence, in the values these labels take. In the first class of formulations, which is similar to models presented in [1] and [2], the label of a node takes the value of the index of the depot of the cycle to which that node belongs. In the second class of formulations, the label of a node takes the value of the number of the cycle it belongs to. We also present valid inequalities for each class of formulations. The main conclusions of this study are that the first class of formulations is preferable to the second in regards to computational times, and the valid inequalities result in lower computational times.

References

[1] Burger M., Su Z., De Schutter B. "A node current-based 2-index formulation for the fixed-destination multi-depot travelling salesman problem", European Journal of Operational Research, **265**, No. 2, 463–477 (2018).

[2] Bektaş T., Gouveia L., Santos D. "Compact formulations for multi-depot routing problems: Theoretical and computational comparisons", Computers & Operations Research, **124**, No. 2, 105084 (2020).

Contributed Talk 9

Dynamics in Random Contact Networks: epidemic propagation in SIS and SIR models

Beatriz Coimbra[1,2*,*], J. Leonel Rocha[1,2], S. Carvalho[1,2]
[1] CEAUL - Universidade de Lisboa
Instituto Superior de Engenharia de Lisboa, ISEL-IPL
[2] Instituto Superior de Engenharia de Lisboa, ISEL-IPL
[*] a47549@alunos.isel.pt

Abstract

The study of dynamic networks attempts to explain the evolution of the diffusion phenomena, in particular the spread of an epidemic. The analysis of the evolution of an event in a network enables well informed decisions in various social contexts. One of these settings may be related to public health, in which we can figure out the right moment to intervene in a target group, applying various measures in order to delay the spread of a certain infectious disease. This work aims to study the propagation dynamics in distinct random networks, using the SIS and SIR models. Therefore, different random networks will be applied to each of the models in order to understand how the spreading evolves. Furthermore, to analyse and simulate how dynamics works in each network, taking into account its parameters such as the probability p, the number of nodes n, the number of neighbours of each node and the importance of the choice of the seeds. It is our purpose to understand which model best represents a real life situation.

References

[1] Barrat A., Barthélemy M. and Vespignani A., Dynamical Processes on Complex Networks, Cambridge University Press, (2008).

[2] Easley D. and Kleinberg J., Networks, Crowds and Markets: Reasoning About a Highly Connected World, Cambridge University Press, (2010).

[3] Kiss I. Z., Miller J. C. and Simon P. L., Mathematics of Epidemics on Networks, Springer International Publishing, (2017).

[4] Macdonald B., Shakarian P., Howard N. and Moores G., "Spreaders in the Network SIR Model: An Empirical Study," ArXiv, **abs/1208.4269**, (2018).

[5] Menczer F., Fortunato S. and Davis C. A., A First Course in Network Science, Cambridge University Press, (2020).

[*]Bolsa de Investigação do CEAUL, Centro de Estatística e Aplicações da Universidade de Lisboa, para Estudante de Mestrado na FCiências.ID - Associação Para a Investigação e Desenvolvimento de Ciências, no âmbito do Projeto UIDP/00006/2020.

Contributed Talk 10

Combining repulsion and attraction at different scales in spatial point processes

Iúri J. F. Correia[1,*], Soraia A. Pereira[1], Tiago A. Marques[1,2]
[1] Centro de Estatística e Aplicações - Universidade de Lisboa, Faculdade de Ciências da Universidade de Lisboa, Portugal,
[2] Centre for Research into Ecological and Environmental Modelling, University of St. Andrews, Scotland
[*] ijcorreia@fc.ul.pt

Abstract

The main goal of this project is to study and model point patterns that exhibit two different types of interactions at different scales. Specifically, how observations interact in space and time for different scales [1], [3]. The data under analysis include active fires in mainland Portugal (FIRMS, NASA) and earthquakes in Azores (ISC). These data consist in point detections and, within the spatial statistics framework, are target to spatial point process statistical methodologies [2], [4], [5]. As a starting point, a brief spatial analysis of those patterns is presented, as well as the most used methods for modelling. Exploratory analysis usually includes functions like the inhomogeneous K-function and the inhomogeneous pair correlation function which are also presented and analysed. Their estimates showed presence of both aggregation and repulsion behaviour at different scales. Future work includes a third case study with caribou sightings in Greenland (GINR). Some spatial point pattern models like empirical models (*e.g.*, Geyer or Matérn-thinned Cox processes) or mechanistic models (*e.g.*, self-exciting and self-correcting processes) that allow capturing this behaviour are also certainly to be considered once a better understanding is achieved concerning the dynamic within the data.

References

[1] Andersen I. T., Hahn U., "Matérn thinned cox processes," Spatial Statistics, **15**, 1–21 (2015).

[2] González, J. A., RodrÃguez-Cortés, F. J., Cronie, O., and Mateu, J., "Spatio-temporal point process statistics: A review," Spatial Statistics, **18**, 505–544 (2016).

[3] Raeisi, M., Bonneu, F., and Gabriel, E., "A spatio-temporal multi-scale model for Geyer saturation point process: Application to forest fire occurrences," Spatial Statistics, **41**, 100492 (2021).

[4] Diggle P. J., Statistical Analysis of Spatial and Spatio-Temporal Point Patterns, Taylor & Francis Inc., (2013).

[5] Baddeley A., Rubak E., and Turner R., Spatial Point Patterns - Methodology and Applications with R, Taylor & Francis Inc., (2015).

Contributed Talk 11

Chiral symmetry-breaking in chemical systems: ODE model and bifurcation analysis

Priscila Costa Diniz[1,*]
[1] Department of Science and Technology, Universidade Aberta
[*]epriscila.diniz@gmail.com

Abstract

Many fundamental molecules for the maintenance of life have a chiral structure, that is, they exist in two mirror-image forms: left-handed and right-handed. Whilst many can be made in both forms, they typically exist in only one form in nature, for example, amino acids are left-handed form, and sugars are right-handed. We are concerned with this chiral symmetry-breaking in chemical systems which can be modeled using ordinary differential equations. We focus on the APED model: Activation-Polymerization-Epimerization-Depolymerization [2]. The emergence of chirality can be described using asymptotic and adiabatic approximations, and through the analysis of the bifurcations in these ODE systems. This presentation consists of showing some results based on the analysis of the APED system [1].

References

[1] Diniz, P. C., Wattis, J. A. D. & da Costa, F. P."Mathematical Models of Chiral Symmetry-breaking - A Review of General Theories, and Adiabatic Approximations of the APED System". Origins of Life and Evolution of Biospheres 52, 183-204. (2022)

[2] Plasson R., Bersini H., Commeyras A. "Recycling Frank: Spontaneous emergence of homochirality in noncatalytic systems". Proc Nat Acad Sci 101:16733-16738. (2004)

Contributed Talk 12

Probability of Collision of satellites and space debris for short-term encounters: Rederivation and fast-to-compute upper and lower bounds

Ricardo Ferreira[1,*], Cláudia Soares[1], Marta Guimarães[2]
[1] NOVA School of Science and Technology
[2] Neuraspace
[*] rjn.ferreira@campus.fct.unl.pt

Abstract

The amount of debris in orbit, especially in Low-Earth Orbit (LEO), has increased significantly over the years. It is estimated that, in orbit, there are millions of fragments a few millimeters in size and thousands of inoperative satellites and discarded rocket stages. Given the high speeds that these fragments can reach, even fragments a few millimeters in size can cause fractures in a satellite's hull or put a serious crack in the window of a space shuttle [1]. With the growing interest in space exploration, the prediction of potential collisions between objects in orbit has become a crucial issue.

The conventional method proposed by Akella and Alfriend in 2000 remains widely used to estimate the probability of collision in short-term encounters [2]. This study introduces a novel derivation based on first principles that naturally allows for tight and fast upper and lower bounds for the probability of collision. We tested implementations of both probability and bound computations with the original and our formulation on a real Conjunction Data Message (CDM) dataset used in ESA's Collision Avoidance Challenge. Our approach reduces the calculation of the probability to two one-dimensional integrals and has the potential to significantly reduce the processing time compared to the traditional method, from 80% to nearly real-time.

References

[1] Pelton, Joseph N and Ailor, WH, Space debris and other threats from outer space, Springer (2013).

[2] Akella, Maruthi R and Alfriend, Kyle T ,"Probability of collision between space objects," Journal of Guidance, Control, and Dynamics, **23**, No. 5, 769–772 (2000).

Contributed Talk 13

Hedging in the Electricity Markets: Adapting to the Growing Role of Renewable Energy

André Folgado[1,2*], Miguel Fonseca[1], Ricardo Covas[2]
[1] Center for Mathematics and Applications, Faculty of Sciences and Technology, Nova University of Lisbon, Portugal
[2] Global Energy Management, EDP
[*] a.folgado@campus.fct.unl.pt

Abstract

Hedging is a crucial risk management tool in the electricity markets that involves reducing the impact of uncertainty in various factors such as interest rates, electricity prices, carbon prices, and foreign exchange (forex) rates. Electricity markets are complex and constantly changing, making it challenging to accurately forecast future prices and manage risks. Therefore, market participants use various hedging strategies to protect themselves from price volatility.

In the electricity markets, hedging involves using financial instruments such as futures, options, and swaps, which enable market participants to lock in prices for future transactions [1]. These instruments are priced based on a range of factors including interest rates, forex rates, electricity and carbon prices

With this work, it is intended develop a model which will be able to address the increasing volatility of the electricity market which is due to the increase of renewable sources. Unlike conventional sources, such as coal or natural gas, renewable energy sources are dependent on weather conditions and are subject to fluctuations in supply. This can lead to sudden spikes or drops in electricity prices, making it more challenging for market participants to forecast and manage risks. In addition, more traditional hedging models function are developed under continuous time assumption whereas the real life decision making within the market happens in a discrete form.

An analysis of various existing models is presented including the most popular ones, such as the Black-Scholes model, a stochastic model that takes into account the volatility of the underlying asset, the time to expiration, and the strike price, and other developed more recently that incorporate other type of dynamics, such as the Heston model or the GARCH model, which incorporate factors such as interest rates and energy prices. [2]

References

[1] Benth F. E., Schmeck M. D., "Pricing Futures and Options in Electricity Markets," in: The Interrelationship Between Financial and Energy Markets, Springer Berlin Heidelberg, Berlin, 2014, pp.223–260.

[2] Hull J. C., Options, Futures, and Other Derivatives, Global Edition, Pearson Education, (2021).

Contributed Talk 14

Finding Real-World Orbital Motion Laws from Data

João Funenga[1,*], Cláudia Soares[1], Henrique Costa[2]
[1] Faculdade de Ciências e Tecnologia, Universidade Nova de Lisboa,
[2] Neuraspace
[*] j.funenga@campus.fct.unl.pt

Abstract

Purpose - Careful planning of propellant used for satellites ensures they follow a set trajectory. Drag is a major force acting on a satellite; if not considered, it can cause position errors of up to hundreds of kilometers. Finding orbital motion laws from data is a critical step in improving predictions of a satellite's position.

Mehodology - We use Sparse Identification of Nonlinear Dynamics (SINDy) to discover equations that describe laws of physics in space that are not deterministic and can be influenced by multiple factors such as drag or the reference area (related to the attitude of the satellite). This aims to find partial differential equations (PDEs) of a physical system using only the system's state over time. Unlike previous works, we maintain the physically interpretable coordinate system and do not perform any dimensionality reduction technique on the data. Finding the interpretable stochastic PDEs from data will unlock many space surveillance and tracking applications.

Results - We have successfully trained SINDy to discover equations that describe an orbit with high-fidelity ephemerides. Training the model with multiple representative trajectories of Low Earth Orbit - with various inclinations, eccentricities, and altitudes - and testing it with unseen orbital motion patterns, we attained a mean error of around 140 km for the positions and 0.12 km/s for the velocities.

Conclusion - Uncovering orbital motion PDEs with SINDy from highly informative, high-entropy datasets provides advantages that traditional models cannot, such as delivering interpretable, accurate, and complex models of orbital motion that can be used for propagation or as inputs to predictive models for other variables of interest, like atmospheric drag or the probability of collision in an encounter of a spacecraft with space objects.

Contributed Talk 15

Heuristics for the Black and White Travelling Salesman Problem

Sofia Henriques[1,2*], Ana Paias[1,2]
[1] DEIO, Faculdade de Ciências, Universidade de Lisboa, C6, Piso 4, Lisboa 1749-016, Portugal
[2] Centro de Matemática, Aplicações Fundamentais e Investigação Operacional, Faculdade de Ciências, Universidade de Lisboa, Portugal
*fc51494@alunos.fc.ul.pt

Abstract

The *Black and White Travelling Salesman Problem* (BWTSP) constitutes a variant of the *Travelling Salesman Problem* (TSP). Similarly to the TSP, it is formally defined on a directed graph with a set of vertices V, a set of arcs A and each arc has an associated cost. Each vertex from V is coloured as either black or white, thus V can be partitioned into two subsets W and B, the former containing all white nodes in V and the latter containing all the black ones. The objective of the BWTSP is to determine the Hamiltonian circuit with minimal cost in the graph which satisfies two conditions: it must not contain more than Q white nodes between two consecutive black vertices, and the total length between two consecutive black vertices must not exceed a value L. The BWTSP has real-life applications in the design of telecommunication networks and in the scheduling of aircraft operations.

In the computational complexity theory, the BWTSP is classified as an NP-hard problem. Our goal in this work was to develop heuristics which can be applied on both symmetric and asymmetric instances of the BWTSP, so that these methods can be used to solve more real-life problems. We proposed and compared three constructive heuristics for the BWTSP. Furthermore, an Iterated Local Search (ILS) algorithm was proposed as an improvement heuristic. In order to justify our choice of parameters for the algorithm, we compared the performance of the ILS considering different combinations of parameters. We also studied in more detail the performance of the ILS we proposed and analysed the differences regarding the quality of the final solutions between symmetric and asymmetric instances.

Keywords: *Black and White Travelling Salesman Problem*, Combinatorial optimization, Heuristic methods, Iterated local search.

References

[1] Henriques, S. "Heuristics for the Black and White Travelling Salesman Problem". [Dissertação de Mestrado em Estatística e Investigação Operacional]. Faculdade de Ciências da Universidade de Lisboa (2022).

Contributed Talk 16

Leveraging Group Equivariance for Trustworthy Machine Learning Models

Diogo Lavado[1,*], Cláudia Soares[1], Alessandra Micheletti[2], Patrizio Frosini[3]
[1] NOVA School of Science and Technology
[2] University of Milan,
[3] University of Bologna
[*] d.lavado@campus.fct.unl.pt

Abstract

Machine learning has become an increasingly popular area of research in mathematics, with applications in a wide range of fields, from computer vision to natural language processing. A key challenge in machine learning is developing models that are both trustworthy and accurate, with the ability to scale to large datasets.

In this work, we take advantage of group equivariant non-expansive operators (GENEOs) [1, 2] as a powerful tool for developing trustworthy and interpretable machine learning methods. GENEOs are operators that act on topological spaces, such as point clouds, and are designed to respect certain symmetries or invariances. In other words, they transform data into higher-level representations while respecting a predefined set of meaningful features, making them particularly suitable for data analysis and feature extraction. We propose a novel white-box model, SCENE-Net, that leverages on GENEOs to provide intrinsic geometric interpretability. SCENE-Net identifies signature shapes in point clouds by transforming them into high-level spaces, where it is easier to distinguish between different objects. We applied our model to a challenging real-world problem, namely the detection of power line supporting towers, to reduce human effort in inspecting large power lines—a key task in the prevention of forest fires and large-scale power outages. SCENE-Net offers robustness to noisy data and low-resource deployment and has on-par performance with state-of-the-art methods.

Overall, our work showcases the potential of GENEOs as a mathematical framework for developing trustworthy and interpretable machine learning. By exploiting equivariance, we can incorporate prior knowledge into our models and gain insights into their decision-making processes. This has significant implications for fields such as autonomous driving and power grid inspection, where transparent machine learning is critical for a responsible deployment. We hope that our work inspires further research into the development of trustworthy machine learning.

References

[1] Bergomi, M.G., Frosini, P., Giorgi, D., Quercioli, N.: Towards a topological–geometrical theory of group equivariant non-expansive operators for data analysis and machine learning. Nature Machine Intelligence **1**(9), 423–433 (2019)

[2] Cascarano, P., Frosini, P., Quercioli, N., Saki, A.: On the geometric and riemannian structure of the spaces of group equivariant non-expansive operators. arXiv preprint arXiv:2103.02543 (2021)

Contributed Talk 17

Air Quality Data Analysis with Symbolic Principal Components

Catarina P. Loureiro[1,*], M. Rosário Oliveira[1], Paula Brito[2], Lina Oliveira[3]
[1] CEMAT & Dep. Mathematics, Instituto Superior Técnico
[2] LIAAD-INESC TEC & Fac. Economia, Universidade do Porto
[3]CAMGSD & Dep. Mathematics, Instituto Superior Técnico
[*]catarinapadrela@tecnico.ulisboa.pt

Abstract

Symbolic data analysis is a growing area of study, that models complex data types like intervals. In this work, we examine air quality data through the lens of symbolic data analysis, following the method proposed in [1]. We start by formulating the interval-valued data framework and estimating the mean and variance via the method of moments. Then the likelihood function is estimated using a bivariate copula to model pairwise dependencies between the interval-valued variables. From there, principal components are obtained and used to fit a generalized extreme value distribution. Finally, the quantiles of this distribution are used to build a control chart and ultimately identify outlying observations.

References

[1] Lin, LC., Guo, M., Lee, S., "Monitoring photochemical pollutants based on symbolic interval-valued data analysis", Advances in Data Analysis and Classification (2022).

Contributed Talk 18

Matérn Correlation and Stochastic Partial Differential Equation

Faustino Maciala[1,*], Raquel Manezes[2]
[1] University of Minho, Instituto Superior Politécnico de Cabinda - ISPCAB
[2] University of Minho, Centre of Mathematics (CMAT)
[*] id10604@alunos.uminho.pt

Abstract

The Modeling of geostatistical processes is a complex problem in spatial statistics. Usually, the maximum likelihood estimation of the model parameters is considered. Since the likelihood involves an unobserved GF (Gaussian Field), one can resort to simulation and numerical techniques to achieve its maximization, typically involving a high computational cost. Alternatively, one can use a numerical method based on the Laplace approximation of the marginal likelihood and adopt an SPDE-based technique to approximate the latent GF ([1], [2], [3], Chapter 11 in [4]). In this work, we propose to study the paper [5], which provides an explicit link between a GF (continuous in space) and a GMRF[†] (discrete in space) as a solution to a (linear) SPDE[‡]. This is possible when a spatial correlation structure in the Matérn class is assumed.

Key Words: Matérn correlation; SPDE.

References

[1] Diggle P. and Giorgi E. "Model-based geostatistics for global public health: Methods and applications," isbn: SS256167318974092, Chapman and Hall/CRC Press (2019).

[2] Dinsdale D. and Salibian-Barrera M. "Methods for preferential sampling in geostatistics," Journal of the Royal Statistical Society (2019), Series C (Applied Statistics), 68(1).

[3] Monteiro A.; Menezes R. and Silva M.E. Modelling preferential sampling in time, Boletín de Estadística e Investigacíon Operativa (2019), series 35(3), 180â€“196.

[4] Zuur A.; Ieno E. and Saveliev A. Beginnerâ€™s Guide to Spatial, Temporal and Spatial Temporal Ecological Data Analysis with R-INLA, Highland Statistics Ltd., (2017).

[5] Lindgren F.; Rue H. and Lindstrom J. "An explicit link between Gaussian fields and Gaussian Markov random fields: the stochastic partial differential equation approach," Journal of the Royal Statistical Society, Serie B (Statistical Methodology), 73, 423-498, (2011).

[†] Gaussian Markov Random Field
[‡] Stochastic Partial Differential Equation

Contributed Talk 19

A functional interpretation over finite types with star types

Lia Malato[1,*],
[1] FCUL and LASIGE, Universidade de Lisboa, Lisboa
[*] lia.malato@hotmail.com

Abstract

Functional interpretations and finite types have first been defined by Gödel in 1958 for arithmetic and were rapidly adapted to other branches such as logic, analysis and set theory. In this talk we extend the work published in [1]. We define a new functional interpretation based on Shoenfield's functional interpretation for classical logic and extend it to finite-order logic by introducing the notion of (finite) star types which represent finite non-empty sets. We give results related to soundness and characterization together with some corollaries and finish off by showing how the characteristic principle of this interpretation, the bounded axiom of choice, can be interpreted.

References

[1] Ferreira. F., Ferreira. G., "A herbrandized functional interpretation of classical first-order logic," Arch. Math. Logic, **56**, 523–539 (2017).

[2] Gaspar. J., "Em torno das interpretações funcionais da aritmética," Master's thesis, Universidade de Lisboa (2016).

[3] Diller. J., Functional interpretations, World Scientific, Münster (2019).

Contributed Talk 20

Using spectral gating noise reduction algorithm to denoise sound data and identify patterns: a narwhal example

Carolina S. Marques[1,*], Emmanuel Dufourq[2,3], Carl Donovan[4,5], Marianne Marcoux[6], Tiago A. Marques[1,4,7]

[1] Centro de Estatística e Aplicações, Universidade de Lisboa, Lisbon, Portugal,
[2] African Institute for Mathematical Sciences, Muizenberg, South Africa
[3] Stellenbosch University, Stellenbosch, South Africa
[4]Centre for Research into Ecological and Environmental Modelling, University of St Andrews, St Andrews, Scotland
[5] DMP Statistical Solutions
[6]Arctic Aquatic Research Division, Fisheries and Oceans Canada, Winnipeg, MB R3T 2N6, Canada
[7]Departamento de Biologia Animal, Universidade de Lisboa, Lisbon, Portugal
*csmarques@fc.ul.pt

Abstract

Sound files containing wildlife sounds for identification typically contain background noise, resulting in low signal to noise ratio which needs denoising prior to analysis. One of the possible approaches is the use of a spectral gating noise reduction algorithm. Spectral gating works by analyzing the frequency content of the audio signal using the Short-Time Fourier Transform (STFT) and comparing it to a noise profile to identify noise bands [1]. A binary mask is then created by comparing the power spectrum of each time-frequency bin to the noise profile, and the binary mask is applied to the original audio signal to selectively attenuate or remove the noise bands. This results in a cleaner and less noisy sound signal. In this study we apply the spectral gating algorithm to identify clicking sounds of the narwhals, *Monodon monoceros*. We applied our approach on data from Acousonde tags attached to 5 narwhals, where we seek to extract their echolocation clicks. The training dataset includes 1.5 hours of recording from 5 tags and our results show that the spectral gating provides excellent denoising of the data , prior to click extraction. We identified approximately 20,000 narwhal sounds and, against a validation dataset of 19 minutes (more than 20% of the original data), we estimate a sensitivity of 0.66. This suggests spectral gating noise reduction algorithm might be a fast and reliable way of cleaning animal borne tag data, which simplifies the extraction of training data for more advanced deep learning methods.

References

[1] Sainburg T. Thielk M. Gentner, T. Q. "Finding, visualizing, and quantifying latent structure across diverse animal vocal repertoires," PLOS Computational Biology, Public Library of Science, No. 10, 1–48 (2020)

Contributed Talk 21

Exploring Complex Interactions in Tree-Mycorrhizal Fungus Networks: Community Analysis and Centrality Measures

João Marques[1,*,§] and J. Leonel Rocha[1,2]
[1] Instituto Superior de Engenharia de Lisboa, ISEL-IPL
[2] CEAUL, Universidade de Lisboa
[*]a50941@alunos.isel.pt

Abstract

Mycorrhizal networks are important underground networks that form between plant roots and mycorrhizal fungi [3]. This structure plays a vital role in nutrient cycling, plant communication and growth, especially in nutrient-poor environments. Graph theory can be used to model and analyze these networks, with plants represented as nodes and their connections through mycorrhizal fungi represented as edges [1, 2]. Scientists can use various graph theoretical parameters, such as degree distribution, clustering coefficient and path length, to gain a better understanding of the network's functioning and the interactions between trees and their associated fungi. In this study, we aim to investigate the complex structure of mycorrhizal networks by using mathematical modeling and graph theory. To understand the organization and function of mycorrhizal networks, we propose community analysis to identify cohesive subgroups of fungal species that are tightly connected within the network. Furthermore, centrality measures such as degree, betweenness, eccentricity and proximity centrality will be used to identify the most important nodes in the network and to gain insights into how the network changes over time. Understanding the complex structure of mycorrhizal networks is critical for improving our knowledge of plant-fungal interactions and may have important implications for managing ecosystems in the future [3].

References

[1] Beiler K. J., Durall D. M., Simard S. W., Maxwell S. A., Kretzer A. M., "Architecture of the wood-wide web: Rhizopogon spp. genets link multiple Douglas-fir cohorts," New Phytologist, **185**, No. 2, 543–553 (2010).

[2] Beiler K. J., Simard S. W., Durall D. M., "Topology of tree-mycorrhizal fungus interaction networks in xeric and mesic Douglas-fir forests," Journal of Ecology, **103**, No. 3, 616–628 (2015).

[3] Simard S. W., Beiler K. J., Bingham, M. A., Deslippe J. R., Philip L. J., Teste F. P., "Mycorrhizal networks: Mechanisms, ecology and modelling," Fungal Biology Reviews, **26**, No. 1, 39–60 (2012).

[§]O relatório *"Introdução ao Estudo das Redes Fúngicas"* foi realizado pelos alunos João Marques, José Casimiro e Rui Nobre na UC de Grafos e Redes do MMAI, ISEL-IPL.

Contributed Talk 22

An interpretable machine learning approach for predicting data centers usage

Gonçalo Mateus[1,2,*], Cláudia Soares[1], João Leitão[1], António Rodrigues[2]
[1] NOVA School of Science and Technology, Lisbon, Portugal
[2] Novobanco, Lisbon, Portugal
[*] gf.mateus@campus.fct.unl.pt

Abstract

In recent years, Cloud computing usage has considerably increased and, nowadays, it is the backbone of many emerging applications. However, behind cloud structures, we have physical infrastructures (data centers) for which managing is difficult due to unpredictable utilization patterns. To address the constraints of reactive auto-scaling, data centers are widely adopting predictive cloud resource management mechanisms. However, predictive methods rely on application workloads and are typically pre-optimized for specific patterns, which can cause under/over-provisioning of resources. Accurate workload forecasts are necessary to gain efficiency, save money, and provide clients with better and faster services.

Working with real data from a Portuguese bank, we propose EAMDrift. This novel method combines forecasts from multiple individual predictors by giving weights to each individual model prediction according to a performance metric. EAMDrift automatically retrains when needed and identifies the most appropriate models to use at each moment through interpretable mechanisms. We tested our novel methodology in a real data problem, by studying the influence of external signals (mass and social media) on data center workloads. As we are working with real data from a bank, we hypothesize that users can increase or decrease the usage of some applications depending on external factors such as controversies or news about economics. For this study, EAMDrift was projected to allow multiple past covariates.

We evaluated EAMDrift in different workloads and compared the results with several baseline methods models. The experimental evaluation shows that EAMDrift outperforms individual baseline models in 15% to 25%. Compared to the best black-box ensemble model, our model has a comparable error (increased in 1-3%). Thus, this work suggests that interpretable models are a viable solution for data center workload prediction.

Contributed Talk 23

Inverse Semigroups vs Orthodox Semigroups

Ana Catarina C. Monteiro[1,*] Gracinda M.S. Gomes[1]
[1] Faculty of Sciences, University of Lisbon,
[*] acatarinacm@gmail.com

Abstract

A class of groups which is closed under taking subgroups, quotients and direct products is said to be a variety of groups. A class of groups which is closed for quotients and group subdirect products is said to be a formation of groups. The study of these particular classes of groups has been extensively explored, see [3] for example.

More recently, Gomes and Nobre introduced the concept of formation for inverse semigroups in [1] deducing several properties, and in particular, relations between classes of inverse semigroups and groups.

As a consequence of ongoing work, we propose an extension of the concept of bivariety introduced in [2] to the concept of formation of orthodox semigroups. In this presentation, inspired by what was done in [1] and after introducing various concepts, we will compare classes of orthodox semigroups, of inverse semigroups and of groups, discussing the property of being formations, or bivarieties or varieties.

References

[1] Gomes G. M.S., Nobre I. J., "Formations of inverse semigroups" Semigroup Forum, 2022.

[2] Kadourek J., Szendrei M. B., "A new approach in the theory of orthodox semigroups" Semigroup Forum, 1990.

[3] Ballester-Bolinches, Ezquerro L. M., Classes of Finite Groups, volume 584 [Springer Science & Business Media], (2006).

Contributed Talk 24

On the Navier-Stokes equations with regularized directional do-nothing open boundary conditions

Pedro Nogueira[1,*], Ana Silvestre[1,2]
[1] CEMAT, Instituto Superior Técnico, Universidade de Lisboa, Portugal
[2]Department of Mathematics, Instituto Superior Técnico, Universidade de Lisboa, Portugal
[*]pedro.mendes.nogueira@tecnico.ulisboa.pt

Abstract

Inspired by the mathematical models proposed in [1, 2, 3, 4], where open Directional-Do-Nothing (DDN) boundary conditions are proposed for the Navier-Stokes equations, we consider a mixed boundary problem with homogeneous-Dirichlet boundary conditions combined with Neumann DDN condition.
Our first approach to this problem [8] replaces the DDN condition by a Regularised-Directional-Do-Nothing (RDDN) condition [6, 5] . This is convenient for the application of Newton method when carrying out numerical simulations and for studying optimal control problems for this Navier-Stokes problem.
In an appropriate functional framework, associated with a saddle point approach similar to the one used in [7], we begin by establishing the well-posedness of the direct problem. Then, we study the convergence of the regularised problem to the problem with DDN condition, including the order of convergence, with some numerical simulations to support the theoretical results.
Lastly, we analyse an optimal control problem of velocity tracking-type by means of a distributed force. We prove the existence of optimal solutions, justify the Gâteaux derivative of the control-to-state map and deduce the first order necessary conditions for optimality.

References

[1] Braack, M., Mucha, P.B., Directional do-nothing condition for the Navier-Stokes equations, *J. Comp. Math.*, **32**(5), 507-521 (2014).

[2] Bruneau, C.-H., Boundary conditions on artificial frontiers for incompressible and compressible Navier-Stokes equations, *ESAIM Math. Model. Numer. Anal.*, **34**(2), 303-314 (2000).

[3] Bruneau, C.-H., Fabrie, P., Effective downstream boundary conditions for incompressible Navier-Stokes equations, *Int. J. Numer. Meth. Fluids*, **19**, 693-705 (1994).

[4] Bruneau, C.-H., Fabrie P., New efficient boundary conditions for incompressible Navier-Stokes equations: a well-posedness result, *RAIRO-MMAN*, **30** (7), 815-840 (1996).

[5] S. Dong, A convective-like energy-stable open boundary condition for simulations of incompressible flows, *J. Comput. Phys.*, **302** (2015), 300-328.

[6] S. Dong, G.E. Karniadakis, C. Chryssostomidis, A robust and accurate outflow boundary condition for incompressible flow simulations on severely-truncated unbounded domains, *J. Comput. Phys.*, **261** (2014), 83-105.

[7] Manzoni, A., Quarteroni, A., Salsa, S., A saddle point approach to an optimal boundary control problem for steady Navier-Stokes equations, *Mathematics in Engineering*, **1**(2), 252-280 (2019).

[8] Nogueira, P., Silvestre, A.L., On the Navier-Stokes equations with regularized directional do-nothing open boundary conditions, in preparation.

Contributed Talk 25

Characterisation of GM-varieties via equivariant cohomology

Javier Orts[1,*], Pedro F. Dos Santos[1] and Carlos Florentino [2]
[1] Instituto Superior Técnico,
[2] Faculdade de Ciências, Universidade de Lisboa
[*] javier.orts@tecnico.ulisboa.pt

Abstract

A *maximal space* is a finite C_2-CW complex X whose cohomology and the cohomology of its fixed points are related by the following equality:

$$\sum_{q=0}^{\dim X^{C_2}} \dim_{\mathbf{F}_2} H^q(X^{C_2};\mathbf{F}_2) = \sum_{q=0}^{\dim X} \dim_{\mathbf{F}_2} H^q(X;\mathbf{F}_2). \tag{3}$$

It is a fact that the action of C_2 on $H^*(X;\mathbf{F}_2)$ is trivial when X is a maximal space.

This class of spaces was generalised by Krasnov in [1] by allowing the action on cohomology to be non-trivial and modifying (3) conveniently to take this into account. The new spaces are called *Galois-maximal spaces*.

In this talk I will present a characterisation of maximal and Galois-maximal spaces in terms of $\mathrm{RO}(C_2)$-graded Bredon cohomology and Borel cohomology based in the work of Clover May [2].

References

[1] Krasnov, V. A., "Harnack–Thom inequalities for mappings of real algebraic vareities", Izv. Akad. Nauk SSSR Ser Mat., **47**, No. 2, 268–297 (1983).

[2] May, C."A structure theorem for $RO(C_2)$-graded Bredon cohomology", Algbr. Geom. Top. **20**, No. 4, 1691–1728 (2020).

Contributed Talk 26

A Hybrid method for the numerical solution of the Inverse Acoustic Transmission Problem

João Paixão[1,2], Pedro Serranho[1,2,3]
[1] Department of Sciences and Technology, Universidade Aberta
[2] CEMAT, Universidade de Lisboa, Portugal
[3] CIBIT, ICNAS, Portugal
* jcpaixao@gmail.com

Abstract

The Inverse Transmission Problem of time-harmonic acoustic scattering by a penetrable object in an homogeneous medium consists in determining the location and eventually some properties of the unknown obstacle from the knowledge of the incident field and the respective far-field data of the scattered wave.

Let $D \subset \mathbb{R}^2$, be a simply connected domain with a C^2 boundary. The direct problem is formulated as follows: For a given incident field u^i, find two fields $u^s \in C^2(\mathbb{R}^2 \setminus \bar{D}) \cap C(\mathbb{R}^2 \setminus D)$ and $u_D \in C^2(D) \cap C(\bar{D})$ that satisfy the Helmholtz equation in their domains and fulfill the transmission boundary conditions. The scattered field u^s must also be a radiating solution.

The asymptotic behavior to infinity of the scattered field u^s is defined in terms of the so called far-field pattern u_∞. Rellich's Lemma shows that for radiating solutions, the far-field pattern uniquely determines the scattered field see, for instance [1]. Isakov in [2] proved the existence and uniqueness of solution for the inverse problem.

We will propose and discuss the extension of the ideas of the hybrid method described in [3] to the transmission problem, therefore combining ideas of decomposition methods and iterative Newton-type iterations. We will consider the method of fundamental solutions to represent the acoustic fields u^s and u_D.

Given an initial approximation of the boundary we first solve an ill conditioned equation to reconstruct the exterior and interior acoustic fields from the far field data in a spirit of a decomposition method and after, similarly to a Newton iterative method, we linearize the boundary condition to get an update of the boundary location to where the boundary condition is satisfied.

References

[1] Colton D., Rainer K., "Inverse Acoustic and Electromagnetic Scattering Theory", Springer, 2013 (3^{rd} edition).

[2] Isakov V., "On uniqueness in the inverse transmission scattering problem", Communications in Partial Differential Equations, No. 15:11, 1565-1586 (2007).

[3] Ivanyshyn O.,Kress R., Serranho P., "Huygen's principle and iterative methods in inverse obstacle scattering", Adv Comput Math, No. 33, 413–429 (2010).

Contributed Talk 27

Child Growth Curve in Sofala - Mozambique and its comparison with other contexts

Neto Pascoal[1,2*], Fernando Sequeira[1,2], Carlos Geraldes[2,3]
[1] Faculdade de Ciências, Universidade de Lisboa (FCUL)
[2] Centro de Estatística e Aplicações da Universidade de Lisboa (CEAUL)
[3] Instituto Superior de Engenharia de Lisboa (ISEL)
[*] polenepascoal@gmail.com

Abstract

The assessment of human growth is an important instrument to understand health, both individually and collectively. The study involves the realization of the child growth curve in Mozambique using $GAMLSS$ models (Generalized Additive Model for Location, Scale and Shape), and eventually, $SITAR$ (Super Imposition by Translation And Rotation) models for estimating height growth velocity, to model longitudinal data of anthropometric and perinatal measurements from birth to 24 months of age. The results would be compared with other growth curves from other regions, through the construction of a hypothesis test for the effect. The selection of children for the study will be carried out at the health units, following the recommendations of WHO. A sample of at least 500 children is estimated to be collected. Intrinsic and extrinsic factors can influence population growth from region to region, calling into question the consistency of current WHO standards. With this, it is expected to adopt reference curves specific to the Mozambican population and it will be important to understand local growth and compare them with other growth curves, [1], [2] and [3].

Keywords: Growth curve; $GAMLSS$ method; *Odds Ratio*; Longitudinal; SITAR

References

[1] M. D. Stasinopoulos, R. A. Rigby, G. Z. Heller, V. Voudouris, F. "Flexible regression and smoothing: using GAMLSS in R' CRC Press, (2017).

[2] T. J. Cole, M. D. Donaldson, Y. Ben-Shlomo, "Sitar–a useful instrument for growth curve analysis," International journal of epidemiology 36, **6**, 1558–1566 (2010).

[3] M. De Onis, C. Garza, C. G. Victora, A. W. Onyango, E. A. Frongillo, J. Martines, "The who multicentre growth reference study: planning, study design, and methodology, Food and nutrition bulletin," 25 (1-suppl-1), S15–S26 (2004).

Contributed Talk 28

Classification of Interval-Valued Data Using Multi-Class Fisher Discriminant Analysis

Diogo Pinheiro[1,*], M. Rosário Oliveira[1,2], Igor Kravchenko[1], Lina Oliveira[1,3]
[1] Department of Mathematics, Instituto Superior Técnico, Universidade de Lisboa, Portugal
[2] CEMAT, Instituto Superior Técnico, Universidade de Lisboa, Portugal
[3] CAMGSD, Instituto Superior Técnico, Universidade de Lisboa, Portugal
[*] diogo.pinheiro.99@tecnico.ulisboa.pt

Abstract

In Data Science, entities are usually described by vectors of single-valued measurements. Symbolic Data Analysis can model more complex data structures such as intervals and histograms that possess internal variability. In this work, we propose an extension of multi-class Fisher Discriminant Analysis [2] to the interval scenario based on Mallows' distance [1] and Moore's algebraic structure [3]. We illustrate this symbolic approach in the context of classification of a real financial dataset with several classes.

References

[1] Irpino A., Verde R., "A new Wasserstein based distance for hierarchical clustering of histogram symbolic data," Data Science and Classification, 185–192 (2006).

[2] Johnson R. A., Wichern D. W., Applied Multivariate Statistical Analysis, Pearson Prentice Hall, Upper Saddle River, NJ (2007).

[3] Moore R. E., Kearfott R. B., Cloud M. J., Introduction to Interval Analysis, Society for Industrial and Applied Mathematics (2009).

Contributed Talk 29

Natural Gas Storage Valuation and Optimisation under Lévy Processes

Bruno Pires[1,2,*]
[1] IST, University of Lisbon
[2] CEMAT, , University of Lisbon
[*]bruno.pires@tecnico.ulisboa.pt

Abstract

Recently, energy markets have been showing high volatility given the effects of the pandemic, the war in Ukraine and extreme cold waves due to climate change. These shocks cause jumps in natural gas prices and these jumps can be captured resorting to Lévy processes. Given Russia's threat of cutting supply of natural gas to the European Union, large amounts of natural gas were stored to face the last winter. Therefore, the natural gas storage and valuation problem, that can be seen as a real options problem, has an increased importance nowadays and an optimal strategy must be found. [1] solved this problem using an Ornstein-Uhlenbeck process for the spot price. More precisely a geometric model is applied, with a compound Poisson process for the jump part. My goal is to fit other types of Lévy processes to the data apart from this one, in order to get a model that better adjusts to reality than this one, using some of the methodologies applied by [2], that are specific to this kind of markets.

References

[1] Safarov, N., Atkinson, C., "Natural gas storage valuation and optimization under time-inhomogeneous exponential Lévy processes" International Journal of Computer Mathematics, 94:11, 2147-2165, DOI: 10.1080/00207160.2016.1168518 (2015).

[2] Benth, F.E., Benth, J.S., Koekebakker, S. Stochastic Modelling of Electricity and Related Markets, World Scientific (2008).

Contributed Talk 30

High-fidelity Interpretable Inverse Rig: An Accurate and Sparse Solution Optimizing the Quartic Blendshape Model

Stevo Rackovic[1,*], Cláudia Soares[2], Dusan Jakovetic[3], Zoranka Desnica[4]
[1] Instituto Superior Técnico, University of Lisbon
[2] NOVA School of Science and Technology, NOVA University of Lisbon
[3] Faculty of Sciences, Novi Sad
[4] 3Lateral studio, Epic Games
[*] stevo.rackovic@tecnico.ulisboa.pt

Abstract

We propose a method to fit arbitrarily accurate blendshape rig models by solving the inverse rig problem in realistic human face animation. The method considers blendshape models with different levels of added corrections and solves the regularized least-squares problem using coordinate descent, i.e., iteratively estimating blendshape weights, following similar directions as [1], yet extending the method to yield significantly superior results. Besides making the optimization easier to solve, this approach ensures that mutually exclusive controllers will not be activated simultaneously and improves the goodness of fit after each iteration. We show experimentally that the proposed method yields solutions with mesh error comparable to or lower than the state-of-the-art approaches [2, 3] while significantly reducing the cardinality of the weight vector (over 20%), hence giving a high-fidelity reconstruction of the reference expression that is easier to manipulate in the post-production manually.

References

[1] Yeongho S., Jaewoo S., Hyunjin K. P., Lewis J. P., Junyong N. "Artist Friendly Facial Animation Retargeting," Association for Computing Machinery (2011).

[2] Cetinaslan O., Orvalho V. "Sketching Manipulators for Localized Blendshape Editing," Graphical Models, **108** (2020).

[3] Rackovic S., Soares C., Jakovetic D., Desnica Z. "A Majorization-Minimization Based Method for Nonconvex Inverse Rig Problems in Facial Animation: Algorithm Derivation," arXiv preprint arXiv:2205.04289 (2022).

Contributed Talk 31

Tropical representations and identities of the stylic monoid

Duarte Ribeiro[1,*], Thomas Aird[2]
[1] Center for Mathematics and Applications (NOVA Math), NOVA SST
[2] Department of Mathematics, University of Manchester
[*] dc.ribeiro@campus.fct.unl.pt

Abstract

The stylic monoid styl_n of finite rank n, introduced by Abram and Reutenauer [1], is a finite quotient of the plactic monoid of rank n, defined by the action of words, over a finite totally ordered alphabet $\mathcal{A}_n$ with n letters, on the left of columns of semistandard Young tableaux, by Schensted left insertion. Its elements can be uniquely identified with so-called N-tableaux, and it is presented by the Knuth relations and the relations $x^2 \equiv x$, with $x \in \mathcal{A}_n$. It is a finite $\mathcal{J}$-trivial monoid, hence, by [2], for some $k \in \mathbb{N}$, it is in $\mathcal{J}_k$, the pseudovariety in Simon's hierarchy of $\mathcal{J}$-trivial monoids which corresponds to the class of all piecewise testable languages of height k, in Eilenberg's correspondence [3]. The pseudovariety $\mathcal{J}_k$ is defined by the equational theory J_k of all identities $u \approx v$ such that u and v share the same subsequences of length $\leq k$. Blanchet-Sadri [4] has shown that J_k is finitely based if and only if $k \leq 3$, while Johnson and Fenner [5] have shown that the variety described by J_k is generated by the monoid $U_{k+1}(S)$ of $(k+1) \times (k+1)$ upper unitriangular matrices with entries in a non-trivial, idempotent commutative semiring S, of which the tropical max-plus semiring $\mathbb{T}$ is an example.

We exhibit a faithful representation of styl_n as a monoid of upper unitriangular matrices over $\mathbb{T}$. Thus, we show that styl_n generates the pseudovariety $\mathcal{J}_n$. From this, we obtain the equational theory of styl_n, show that it is finitely based if and only if $n \leq 3$, and that its identity checking problem is decidable in linearithmic time. We also solve the finite basis problem for styl_n with involution.

References

[1] Abram, A., Reutenauer, Ch., "The stylic monoid", Semigroup Forum, **105**, No. 1, 1–45 (2022).

[2] Simon, I., Hierarchies of Events with Dot-Depth One, Ph.D. thesis, University of Waterloo, Waterloo (1972).

[3] Eilenberg, S., Automata, Languages, and Machines, Vol. B., Pure and Applied Mathematics, vol. 59, Academic Press, Harcourt Brace Jovanovich, New York (1976).

[4] Blanchet-Sadri, F. "Equations and monoid varieties of dot-depth one and two", Theoret. Comput. Sci., **123**, No. 2, 239–258 (1994).

[5] Johnson, M., Fenner, P., "Identities in unitriangular and gossip monoids", Semigroup Forum, **98**, No. 2, 338–354 (2019).

Contributed Talk 32

Assymptotic Approximations of European Options Prices under the α-Hypergeometric Model

Frederico Rodrigues[1,*]
[1] Faculdade de Ciências, Universidade de Lisboa
*frodrigues26@icloud.com

Abstract

We discover an unique invariant measure for the volatility driver of the α-hypergeometric model. Then, following known results of perturbation theory which can be found in [1], we developed simple first and second order approximations for both put option prices and implied volatility curves in this model with full generality, extending and simplifying the works of [4].

References

[1] Jean-Pierre Foque, George Papanicolaou, Ronnie Sircar, Knut Solna. *Multiscale Stochastic Volatility for Equity, Interest Rate and Credit Derivatives*, Cambridge University Press, 2011.

[2] Claude Martini, José Da Fonseca. "The α-Hypergeometric Stochastic Volatility Model". Em: *Preprint arXiv:1409.5142*, 2014.

[3] Ankush Agerwal, Sandeep Juneja, Ronnie Sircar. "American Options under Stochastic Volatility: Control Variates, Maturity Randomization & Multiscale Asymptotics". Em: *Quantitative Finance, volume 16, issue 1, p. 17 - 30*, 2016.

[4] Nicolas Privault, Qihao She. "Option pricing and implied volatilities in a 2-hypergeometric stochastic volatility model" Em: *Applied Mathematics Letters Volume 53, March 2016, Pages 77-84*, 2016.

[5] Peter Carr. "Randomization and the American Put". Em: *The Review of Financial Studies, volume 11, issue 3, p. 597 - 626*, 1996.

[6] Andrei D. Polyanin, Valentin F. Zaitsev. *Handbook of Exact Solutions for Ordinary Differential Equations, Second Edition* , Chapman & Hall/CRC, 2003.

[7] W. Feller. "Two Singular Diffusion Problems". Em: *Annals of Mathematics 54, Pages 173-182*, 1951.

[8] S. Heston. "A Closed-Form Solution for Options with Stochastic Volatility with Applications to Bond and Currency Options". Em: *Review of Financial Studies 6*, 1993.

[9] J. Hull. *Options, Futures and Other Derivatives, fifth ed.* , Prentice-Hall, 2002.

[10] F. Black, M. Scholes. "The Pricing of Options and Corporate Liabilities". Em: *Journal of Political Economy, 81, Pages 637-654*, 1973.

Contributed Talk 33

A local characterization of quasi-crystal graphs

Inês Rodrigues[1,*], Alan J. Cain[1], António Malheiro[1], Fátima Rodrigues[1]
[1] Center for Mathematics and Applications (NOVA Math), NOVA School of Science and Technology,
*ima.rodrigues@fct.unl.pt

Abstract

The hypoplactic monoid (the monoid of quasi-ribbon tableaux), first introduced by Krob and Thibon [3] and studied in depth by Novelli [4], is an analogue of the classical plactic monoid (the monoid of Young tableaux), appearing in the context of quasi-symmetric functions and non-commutative symmetric functions. Cain and Malheiro [2] introduced a purely combinatorial quasi-crystal structure for the hypoplactic monoid, where words are equal in the monoid precisely when they are in the same position in isomorphic connected components of a quasi-crystal graph.

We provide a local characterization of quasi-crystal graphs, by presenting a set of local axioms, similar to the ones introduced by Stembridge [5] for crystal graphs of type A (or, more generally, for simply-laced root systems). We then prove that quasi-crystal graphs satisfying these axioms are closed under the tensor product recently introduced by Cain, Guilherme and Malheiro [1] and conclude that each connected component of such a graph has a unique highest weight element, whose weight is a composition, and it is isomorphic to a quasi-crystal graph of semistandard quasi-ribbon tableaux.

Acknowledgements: This work is funded by national funds through the FCT - Fundação para a Ciência e a Tecnologia, I.P., under the scope of the projects UIDB/00297/2020 and UIDP/00297/2020 (Center for Mathematics and Applications)

References

[1] Cain A.J., Guilherme R.P., Malheiro A. "Quasi-crystals for arbitrary root systems and associated generalizations of the hypoplactic monoid". arXiv:2301.00271.

[2] Cain A.J., Malheiro A. "Crystallizing the hypoplactic monoid: from quasi-Kashiwara operators to the Robinson–Schensted–Knuth-type correspondence for quasi-ribbon tableaux", J. Algebr. Comb. **45** (2), 475–524 (2017).

[3] Krob D., Thibon J.-Y. "Noncommutative symmetric functions. IV: Quantum linear groups and Hecke Algebras at $q = 0$", J. Algebr. Comb. **6** (4), 339 – 376 (1997).

[4] Novelli J.-C. "On the hypoplactic monoid", Discrete Math., **217** (1–3), 315–336 (2000).

[5] Stembridge J.R. "A local characterization of simply-laced crystals", Trans. Am. Math. Soc. **355** (12), 4807–4823 (2003).

Contributed Talk 34

On the use of graph theory and machine learning algorithms in anti-money laundering systems

Mafalda Sá Ferreira[1,*], Regina Bispo[1,2]
[1] NOVA School of Science and Technology, NOVA University of Lisbon
[2] Center for Mathematics and Applications (NOVA Math) NOVA School of Science and Technology, NOVA University of Lisbon
[*] msm.ferreira@campus.fct.unl.pt

Abstract

Anti-Money Laundering consists of a set of actions aiming at preventing the movement of illegally obtained money through the financial system. The procedures adopted by financial institutions to detect suspicious activities are typically rule-based, resulting in high false positive rates. To improve the effectiveness of these systems, we propose a graph-based approach that incorporates the transactional relationships between clients [1]. New features are computed via random walks and used as input in machine learning methods [2], to build a fraud prediction classification model. To analyze the graph information in a financial context, we aim at building an R package with a synthetic data set that characterizes real relationships and behaviour between clients. We describe all the concepts and methods needed for this study, as well as the data set. In this talk, we provide some preliminary results of the proposed approach.

References

[1] Eddin A. N., et al. "Anti-Money Laundering Alert Optimization Using Machine Learning with Graphs" (2021).

[2] Oliveira C., et al. "GuiltyWalker: Distance to illicit nodes in the Bitcoin network" (2021).

Contributed Talk 35

Variational methods for an optimal partition problem with volume constraint

Makson Santos[1,*], Pêdra Andrade[1], Ederson Moreira dos Santos[2], Hugo Tavares[1]
[1] Instituto Superior Técnico - University of Lisbon
[2] ICMC - University of São Paulo
[*] makson.santos@tecnico.ulisboa.pt

Abstract

We study the existence and regularity of optimal partitions for a problem with volume constrain. In particular, we prove that an optimal partition is connected and the eigenfunction associated with each set is locally Lipschitz continuous, which implies the that the optimal sets are at least open sets. We show that there is a variational formulation to our problem that does not involve subsets, only functions, and we prove the desired properties for the minimizers. See for intance [1] and [2].

References

[1] Briançon T., Hayoun M., Pierre M., "Lipschitz continuity of state functions in some optimal shaping," Calc. Var. Partial Differential Equations, 23(1):13-32, (2005).

[2] Dorin B., Butazzo G. "Variational methods in shape optimization problems", volume 65 of Progress in Nonlinear Differential Equations and their Applications. Birkhäuser Boston, Inc., Boston, MA, (2005).

Contributed Talk 36

'Provability Implies Provable Provability' in FLINSPACE

Paulo Guilherme Santos[1,2,*], Isabel Oitavem[2], Reinhard Kahle[1]
[1] Carl Friedrich von Weizsäcker-Zentrum, Universität Tübingen,
[*] pgd.santos@campus.fct.unl.pt
[2] NOVA-Math, NOVA School of Science and Technology

Abstract

We develop a theory of arithmetic for the class FLINSPACE (this class is the same as Grzegorczyk's class $\mathcal{E}^2$) that we call G_2 (similar to $\mathsf{I}\mathcal{E}^2_*$). We explore connections between G_2 and $\mathsf{I}\Delta_0$, and their implications in the study of complexity classes. After that, we express the usual metamathematical notions in G_2: we define numerations of the axioms of a theory; we define the *standard proof predicate* $\mathtt{Prf}_\xi(x,y)$ that expresses "y is the code of a proof of the formula coded by x according to the numeration ξ"; and we define the *standard provability predicate* $\mathtt{Pr}_\xi(x) := \exists y.\mathtt{Prf}_\xi(x,y)$. We study the uniform derivability condition $\mathtt{Pr}_\xi(x) \to \mathtt{Pr}_\xi(\ulcorner \mathtt{Pr}_\xi(\dot{x}) \urcorner)$. We prove that if $\mathtt{Pr}^{\mathcal{S}}(x)$ is a provability predicate for a finite set of axioms $\mathcal{S}$ (including a finite number of logical axioms), then $\mathsf{G}_2 \vdash \mathtt{Pr}^{\mathcal{S}}(x) \to \mathtt{Pr}_\xi(\ulcorner \mathtt{Pr}_\xi(\dot{x}) \urcorner)$. Moreover, if G_2 can verify its axioms, in the sense that, for a suitable G_2-function $\mathtt{verifier}$, $\mathsf{G}_2 \vdash \xi(x) \to \mathtt{Prf}_\xi(\ulcorner \mathtt{Pr}_\xi(\dot{x}) \urcorner, \mathtt{verifier}(x))$, then $\mathsf{G}_2 \vdash \mathtt{Pr}_\xi(x) \to \mathtt{Pr}_\xi(\ulcorner \mathtt{Pr}_\xi(\dot{x}) \urcorner)$. We also study form of internal Σ_1-completeness for G_2. Finally, we present conditions for a numeration ξ_0 of a finitely axiomatizable theory to satisfy $\mathsf{G}_2 \vdash \mathtt{Pr}_{\xi_0}(x) \to \mathtt{Pr}_\xi(\ulcorner \mathtt{Pr}_{\xi_0}(\dot{x}) \urcorner)$.

References

[1] A. Das, and I. Oitavem, A recursion-theoretic characterisation of the positive polynomial-time functions, 27th EACSL Annual Conference on Computer Science Logic (CSL 2018). Schloss Dagstuhl-Leibniz-Zentrum fuer Informatik.

[2] F. Ferreira, Provably total primitive recursive functions: theories with induction, Contemporary Mathematics, W. Sieg (ed.) Vol. 106. AMS, pp. 137–156, 1990.

[3] T. Strahm, Theories with self-application and computational complexity, Information and Computation, Vol. 185(2), pp. 263–297, 2003.

Contributed Talk 37

Critical Lane–Emden systems with Neumann boundary conditions: existence, regularity, and symmetries

Delia Schiera[1,*], Angela Pistoia[2], Hugo Tavares[1]
[1] Instituto Superior Técnico, Universidade de Lisboa
[2] Sapienza Università di Roma
[*] delia.schiera@tecnico.ulisboa.pt

Abstract

I will present some recent results for a Lane-Emden system on a bounded regular domain with Neumann boundary conditions and critical nonlinearities. We show that, under suitable conditions on the exponents in the nonlinearities, least-energy (sign-changing) solutions exist. In the proof we exploit a dual variational formulation which allows to deal with the strong indefinite character of the problem, and we establish a compactness condition which is based on a new Cherrier type inequality. We then prove such condition by using as test functions the solutions to the system in the whole space and performing delicate asymptotic estimates. I will also briefly present regularity results and some symmetry/symmetry breaking phenomena.

Contributed Talk 38

On the Extension of the Reverse Hölder Inequality for Power Functions on the Real Axis

Alina Shalukhina[1,*]
[1] Faculdade de Ciências e Tecnologia, Universidade Nova de Lisboa,
[*]a.shalukhina@campus.fct.unl.pt

Abstract

We consider the class of all nonnegative on $\mathbb{R}_+$ functions such that each of them satisfies the reverse Hölder inequality uniformly over all intervals with some constant, the minimum value of which can be regarded as the corresponding "norm" of a function. We compare this "norm" with the "norm" of the even extension of a function from $\mathbb{R}_+$ on $\mathbb{R}$. In this work, an upper estimate for the ratio of such "norms" has been obtained. For the special case of power functions on $\mathbb{R}_+$, we give the precise value of the "norm" increase caused by even extension. This value is a lower estimate for the analogous quantity in the case of arbitrary functions. It has been shown that the obtained upper and lower estimates for the general case are asymptotically sharp.

References

[1] Hardy G. H., Littlewood J. E., and Pólya G., Inequalities, Cambridge University Press, Cambridge (1934).

[2] Korenovskii A., Mean Oscillations and Equimeasurable Rearrangements of Functions, Springer-Verlag, Berlin, Heidelberg, and New York (2007).

Contributed Talk 39

Pointwise convergence properties of infinitely many fermions

Neal Bez[1], Shinya Kinoshita[2], Shobu Shiraki[3,*],
[1] Saitama University
[2] Tokyo Institute of Technology
[3] Instituto Superior Técnico
[*] shobushiraki@tecnico.ulisboa.pt

Abstract

Let $d \in \mathbb{N}$ be the spatial dimension. The Schrödinger equation is a fundamental equation in quantum mechanics and particularly important to understand the physics of tiny particles, such as electrons. Its solutions may be written by using the Fourier transform. One of the fundamental questions from the analysis point of view is the pointwise convergence of the solutions as time tends to 0, namely; what is the smallest $s \geq 0$ that guarantees $\lim_{t\to 0} U_m f(x,t) = f(x)$ almost everywhere for all $f \in H^s(\mathbb{R}^d)$? Here, $m \in (0,\infty)\backslash\{1\}$, $U_m f$ denotes the solution to the (more general) fractional Schödinger equation with its initial data f, and Sobolev space $H^s(\mathbb{R}^d)$ (small s means f is less smooth). This famous problem is called Carleson's problem, named after a mathematical giant who initiated it and has led the modern harmonic analysis to develop significantly. As a result, the non-trivial $\frac{d}{2(d+1)}$ is the essential threshold of s (e.g. [3, 4]).

Very recently, [1] came up with a natural extension of this problem for (possibly infinitely) many particles, while the above is only concerned with a single particle. The key to solving the problem is the following (local) maximal inequality for orthonormal systems, which was originally considered as Strichartz-type estimates (in general) by [5] to study the behavior of fermions;

$$\left\| \sup_{t\in(-1,1)} \sum_j \lambda_j |U_m f_j|^2 \right\|_{L^1(\mathbb{R}^d)} \lesssim \|\lambda\|_{\ell^\beta} \tag{4}$$

for all orthonormal system $(f_j)_j$ in $H^s(\mathbb{R}^d)$ and sequences $(\lambda_j)_j \in \ell^\beta$. The new parameter $\beta \in [1,\infty]$ coincides with 1 for the single-particle case but may increase, in general, depending on s. We, for instance, establish (4) for all relevant m and s in one dimension [2].

References

[1] N. Bez, S. Lee, S. Nakamura, *Maximal estimates for the Schrödinger equation with orthonormal initial data*, Selecta Math. **26** (2020), Article 52.

[2] N. Bez, S. Kinoshita, S. Shiraki, *Boundary Strichartz estimates and pointwise convergence for orthonormal systems*, in preparation.

[3] J. Bourgain, *A note on the Schrödinger maximal function*, J. Anal. Math. **130** (2016), 393–396.

[4] X. Du, R. Zhang, *Sharp L^2 estimate of Schrödinger maximal function in higher dimensions*, Ann. of Math. **189** (2019), 837–861.

[5] R. Frank, M. Lewin, E. H. Lieb, R. Seiringer, *Strichartz inequality for orthonormal functions*, J. Eur. Math. Soc. **16** (2014), 1507–1526.

Contributed Talk 40

A Schur ring approach to supercharacters of groups associated with finite radical rings

Tânia Z. Silva[1,*], Carlos A. M. André[1], Pedro J. Freitas[1]
[1] CELC - Faculdade de Ciências, Universidade de Lisboa
[*]fc26785@alunos.ciencias.ulisboa.pt

Abstract

Following [1], we consider the central Schur ring associated with the standard supercharacters of the adjoint group $G(\mathcal{A})$ of a finite radical ring $\mathcal{A}$, and define supercharacters of the subgroup $C_{G(\mathcal{A})}(\sigma)$ consisting of elements fixed by an involution of G that can be defined when $\mathcal{A}$ is endowed with an (anti)involution and has odd characteristic. In particular, we extend known results for unipotent subgroups of the classical finite Chevalley groups that can be found in [2].

References

[1] Hendrickson A. O. F., "Supercharacter theory constructions corresponding to Schur ring products," Communications in Algebra, **40**, No. 12, 4420–4438 (2012).

[2] André C. A. M., Freitas P. J., Neto A. M., "A supercharacter theory for involutive algebra groups," Journal of Algebra, **430**, 159–190 (2015).

Contributed Talk 41

Pricing Renewable Energy Certificates

Vasco Tavares[1,*], João Guerra[1]
[1] ISEG - Lisbon School of Economics and Management, University of Lisbon
[*] vctavares@aln.iseg.ulisboa.pt

Abstract

In this talk, we intend to present a valuation method for Renewable Energy Certificates (RECs). These are designed to promote the production of energy by renewable sources, establish targets for a renewable energy growth, and ultimately to mitigate the effects of climate change.

In this model, we address the valuation of RECs as the solution to a coupled system of Forward-Backward Stochastic Differential Equations (FBSDEs), as in [1]. It contains two stochastic factors: the accumulated RECs sold by an authorized generator and the natural logarithm of the renewable energy production rate. Therefore, two forward Stochastic Differential Equations (SDEs) are considered: one for the renewable generation rate and another one for the accumulated RECs. The backward SDE corresponds to the REC price and can be formulated in terms of a nonlinear Partial Differential Equation (PDE). We discuss the circumstances under which we can ensure the existence and uniqueness of a solution, following a result from [2].

One particular feature of this work is the resulting PDE. It is a convection dominated PDE and contains a nonlinear convective term, which adds some difficulties in finding its numerical solution. In this talk, we intend to present some preliminary results of the implementation of this method by using a numerical scheme presented in [1].

References

[1] Baamonde-Seoane M. A., Calvo-Garrido M. C., Coulon M., Vázquez C., "Numerical solution of a nonlinear PDE model for pricing Renewable Energy Certificates (RECs)", Applied Mathematics and Computation, **404**, No. 126199 (2021).

[2] Bento A. M., Forward Backward Stochastic Differential Equations and Pricing in Emission Markets, Master's Disseration, ISEG-UL, Lisbon (2022).

Contributed Talk 42

Gaussian Processes for Shape Modelling: a Probabilistic Registration Approach

Filipa Valdeira[1,*], Ricardo Ferreira[2], Alessandra Micheletti[3], Cláudia Soares[1]
[1] NOVA School of Science and Technology, Universidade NOVA de Lisboa
[2] μRoboptics
[3] Università degli Studi di Milano
[*] f.valdeira@campus.fct.unl.pt

Abstract

Shape models find increasing applications in fields ranging from medicine to security or animation. Gaussian Processes (GP) have emerged as a powerful framework for developing 3D shape models and performing related tasks in a unified manner [1]. However, existing methods rely on the availability of a large and complete dataset, which can be costly and time-consuming to acquire, often requiring extensive manual input and application-specific pre-processing methods. A more common setting is a dataset with large regions of missing data and outliers, where standard techniques have limited performance. In order to overcome this, we formulate the shape fitting and registration problem as a multi-annotator Gaussian Process Regression. We then prove that the update equations are equivalent to those in state-of-the-art probabilistic registration methods [2], under a small set of assumptions. This allows us to establish a principled update for the unknown parameters of the multi-annotator model. The achieved method (SFGP) [3] shows better performance when dealing with extensive areas of missing data when compared to a state-of-the-art registration method and current approaches for registration with pre-existing shape models.

References

[1] Lüthi M., Gerig T., Jud C., Vetter T. "Gaussian Process Morphable Models" IEEE Transactions on Pattern Analysis and Machine Intelligence, **40**, No. 8, 1860-1873 (2018).

[2] Hirose O. "A Bayesian Formulation of Coherent Point Drift" IEEE Transactions on Pattern Analysis and Machine Intelligence, **43**, No. 7, 2269-2286 (2021).

[3] Valdeira F., Ferreira R., Micheletti A., Soares C. "Probabilistic Registration for Gaussian Process 3D shape modelling in the presence of extensive missing data" SIAM Journal on Mathematics of Data Science, *in press* (2023).

Contributed Talk 43

Inclusion of the ideal of compact operators into the algebra of Wiener-Hopf operators on variable Lebesgue spaces

Márcio Valente[1,*],
[1] Nova School of Science and Technology, Universidade Nova de Lisboa
[*] mac.valente@campus.fct.unl.pt

Abstract

Let $\mathcal{B}_M(\mathbb{R})$ stand for the set of all variable exponents $p(\cdot) : \mathbb{R} \to [1, \infty]$ such that

$$\operatorname{essinf}_{x\in\mathbb{R}} p(x) > 1, \quad \operatorname{esssup}_{x\in\mathbb{R}} p(x) < \infty,$$

and the Hardy-Littlewood maximal operator is bounded on the variable Lebesgue space $L^{p(\cdot)}(\mathbb{R})$. Given $p(\cdot) \in \mathcal{B}_M(\mathbb{R})$, we define the algebra of continuous Fourier multipliers $C_{p(\cdot)}(\dot{\mathbb{R}})$ as the closure of the set of continuous functions on $\dot{\mathbb{R}} := \mathbb{R} \cup \{\infty\}$ with bounded variation with respect to the norm in the space of Fourier multipliers $M_{p(\cdot)}$. Furthermore, the algebra generated by Wiener-Hopf operators with continuous symbols will be denoted by $\operatorname{alg} W(C_{p(\cdot)}(\dot{\mathbb{R}}))$. We show that for all $p(\cdot) \in \mathcal{B}_M(\mathbb{R})$, the ideal of compact operators $\mathcal{K}(L^{p(\cdot)}(\mathbb{R}^+))$ is contained in the algebra $\operatorname{alg} W(C_{p(\cdot)}(\dot{\mathbb{R}}))$.